EXHIBIT C

Acoustic Noise Reduction in Sinusoidal PWM Drives Using a Randomly Modulated Carrier

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***Abstract*—Acoustic noise in an inverter-driven electric machine can be reduced by avoiding the concentration of harmonic energy in distinct tones. One method to spread out the harmonic spectrum without the use of programmed PWM is to cause the switching pattern to be random. It is proposed that the switching pattern can be randomized by modulating the triangle carrier in sinusoidal PWM with band-limited white noise. All the advantages of sinusoidal PWM are preserved with this technique. These include, real-time control, linear operation, good transient response and a constant average switching frequency. By controlling the bandwidth and rms value of the bandwidth limited noise modulation, it is shown that the instantaneous variation in switching frequency as well as the bandwidth of the energy spectrum in the machine can be specified within predetermined limits. Experimental results show the absence of acoustic noise concentrated at specific tones which is present with conventional sinusoidal modulation.**

I. Introduction

Modern three phase motor drives in the medium to high power range typically switch at frequencies in the audio range. That is, above the fundamental frequency of the output voltage (6-step mode) and below 2 to 3 kHz. Acoustic noise in an electric machine is for the most part proportional to the square of the air gap flux density. The majority of the noise is then generated by harmonic flux waves interacting with the fundamental flux wave. The maximum noise, certainly from a psychological standpoint, is generated when the harmonic flux produced is concentrated at a particular frequency such as the switching frequency. It is desirable then to spread out the frequency spectrum of the harmonic voltage, and therefore current, in such a way as to eliminate the presence of specific tones.

A typical six-pulse bridge inverter feeding an induction machine is illustrated in Fig. 1. Certain types of switching regulators, such as the hysteresis current regulator [1], accomplish this spreading of the voltage spectrum inadvertently. The hysteresis regulator has a varying switching frequency which results in a voltage spectrum that does not have a large component at a particular frequency. However, the switching pattern, in the steady-state, is still

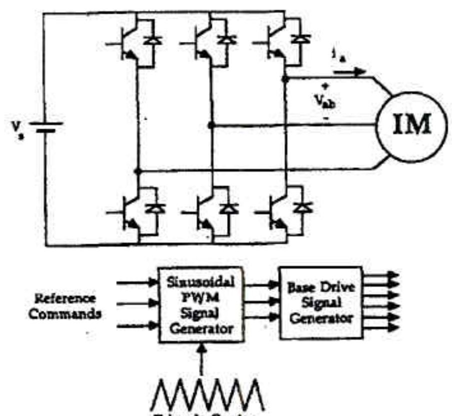

Fig. 1. Schematic for sinusoidal PWM voltage-sourced inverter feeding an induction motor load.

periodic since the switching instances are directly related to the current error. Therefore, the amount of acoustic noise generated depends on the nature of the load. The hysteresis regulator also has the problem of a widely varying switching frequency and low frequency beating is observed under light load conditions.

Alternative methods for acoustic noise reduction have been implemented through the use of harmonic control in programmed PWM schemes [2], [3]. In programmed PWM the switching frequency is maintained constant. The individual tones produced by the switching process are kept low by control of the switching angles. The problem with this approach is that discrete tones in the voltage spectrum still exist since the switching function is periodic. Also, programmed PWM schemes require many waveforms to be stored in memory for varying values of output fundamental magnitude. More critically, since programmed PWM stores the switching angles for a complete cycle of the output voltage it is only useful for acoustic noise reduction when the drive is operating in the steady state. The basic concept of programmed PWM only applies to steady state periodic waveforms. Programmed PWM is not applicable to current regulated drives which are quite often operating in the transient state. Pro-

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grammed PWM results in a slow transient response and somewhat nonlinear relationship of the output voltage with respect to the reference.

II. Modulated Carrier Sinusoidal PWM

An improved PWM scheme with acoustic noise reduction then must have a spectrum without energy concentrated at discrete frequencies and still not have a widely varying switching frequency. Real time control and a linear transfer function are also requisite features. In addition, the spectral energy fed into the motor load should be limited to a defined range. In that way, current in the machine at frequencies significantly below the switching frequency can be avoided. The reduction of machine-generated acoustic noise has been demonstrated with the regularly sampled sigma-delta modulator [4]. The acoustic noise reduction is attributed to the fact that the energy spectrum for this type of controller is spread out. The spectrum is spread in the sigma-delta modulator because the combination of the quantizer and the regular sampling generates a random switching pattern. The switching pattern is random since the error polarity, which is the output of the quantizer, and the sampling instant are not correlated. The concept of randomizing repetitive phenomena to reduce acoustic noise has been widely used for applications in other disciplines. An example is the random spacing or orientation of fan or propeller blades. Placement of gear teeth is also randomized to reduce noise. The same concept of randomizing a repetitive precess can be applied to PWM drives. The limitation of this type of modulation is the fact that the switching instants are discrete. Therefore, higher switching frequencies are required to achieve performance equivalent to PWM.

Reference [6] introduces another method for spreading the inverter output voltage spectrum to reduce acoustic noise. In this approach a random switching function is intentionally created to reduce acoustic noise. With this technique the average value of the regulator reference voltage, over a particular interval, T_n, is found using a triangle comparison. This average is denoted as V^*_{avg}. The T_n interval is divided up into r subintervals. For each of these subintervals, a uniformly distributed random number with expected value V^*_{avg} is generated. The inverter is switched high (or low) at each of the subintervals, depending on whether or not the value of the random number is above (or below) V^*_{avg}. This causes the energy spectrum of the unwanted components of output voltage to be almost flat. This scheme also has the advantage of real-time control. However, the requisite switching frequency for satisfactory performance is even higher than that necessary in discrete pulse switching. In [6], for a 60-Hz fundamental frequency of the output voltage, the switching frequency is 320 kHz.

Sinusoidal PWM has the desired feature of being a real-time linear amplifier with a constant switching frequency. The switching pattern for sinusoidal PWM is illustrated in Fig. 2. The pole (phase) switching function results from

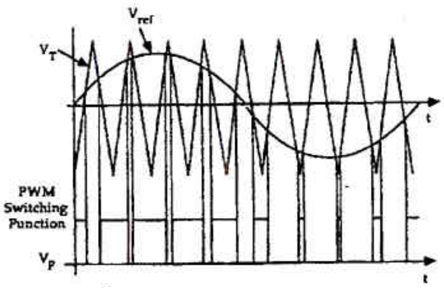


Fig. 2. Waveforms illustrating triangle carrier, reference wave and resulting pole switching function for conventional sinusoidal PWM.

the comparison of a voltage reference wave with a triangular carrier. The resulting output ac load voltage spectrum does, however, have a large component at the switching frequency which results in a tone that is radiated acoustically by the machine. The switching frequency, of course, is the frequency of the carrier or triangle wave, denoted ω_c. The spectrum of the resulting PWM voltage, V_p, is related to the carrier wave, V_T. For example, consider the case of zero modulation. Here V_p is a square wave which results from comparing V_c with zero, and is simply the derivative of the triangle carrier. It is proposed that modulating the carrier frequency in an appropriate manner will spread the energy that would otherwise exist in a tone at ω_c.

To maintain the linearity of the amplifier, it is necessary for the carrier wave to still have straight line segments, i.e., it must remain triangular. As in unmodulated sine-triangle PWM, it is assumed that the instantaneous carrier frequency is always much larger than the frequency of the reference, f_o. Then the duty cycle generated by the PWM comparison is maintained proportional to the value of the reference over one period of the carrier. However, if the slope of subsequent segments are varied, linearity is maintained. Let the instantaneous value of the carrier frequency, i.e., the frequency of a single straight line segment between time t_k and t_{k+1}, be given by

$$f_i = \frac{1}{T_i} = \frac{1}{t_{k+1} - t_k} = \frac{1}{T_c + T_n} \tag{1}$$

and

$$T_n = V_{max} n(t_k) \tag{2}$$

where T_n is the change in the period of any given segment due to the modulation, $n(\bullet)$ is the modulating function, and T_c is the average period of the carrier. This type of modulation is often referred to as frequency shift keying (FSK). Fig. 3 illustrates the carrier waveform.

It is clear that by restricting the magnitude of $n(t)$ in this scheme the switching frequency can be maintained within a predetermined range. It is also very important to constrain the spectrum of the output voltage, V_p, so that

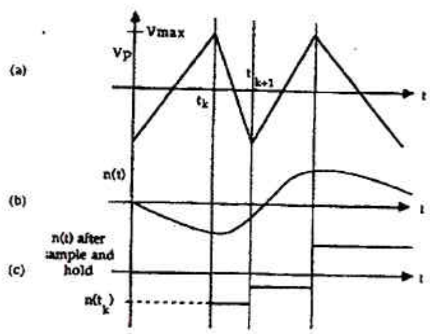


Fig. 3. Waveforms illustrating: (a) Triangle carrier with FSK-type modulation. (b) Modulating function. (c) Modulation function after sample and hole which defines slope of triangle.

substantial low frequency content does not exist. One approach would be to sinusoidally modulate the triangle wave around ω_c. If the magnitude of $n(t)$ is sufficiently small it can be surmised from frequency modulation theory that the bandwidth of the output voltage can be controlled. However, because of the low regular manner in which f_i changes, the tone is still detected. The acoustic noise produced by the machine sounds like a fast-changing tone.

III. Random Modulation

A better approach to sinusoidal modulation is a random modulation of the slope of the carrier waveform. Let the modulating function, $n(t)$, be a random function, say, Gaussian "white" (i.e., wideband) noise. Modulating with very high bandwidth noise results in a large instantaneous change in the triangle frequency. This eliminates the concentration of energy in a tone. It also results in a very wide spectrum for the output voltage, including substantial content at low frequencies which will cause low frequency currents to flow in the machine.

Since the fundamental component of the triangle wave is the primary source of acoustic noise, the higher frequency harmonics can be neglected. Let V_c be the output voltage without the modulated carrier. Then, ignoring components other than a single tone,

$$V_c(t) = A \cos \omega_c t. \tag{3}$$

If the modulating function has a narrow bandwidth, the spectral content of V_p, the output voltage, can be constrained within a specified range of frequencies. For the sake of analysis then, assume that the modulation is continuous. Therefore,

$$V_p(t) = A \cos [\omega_c t + \mu(t)] \tag{4}$$

where $\mu(t)$ is the change in instantaneous angle due to modulation. The autocorrelation function is

$$\begin{aligned} R(t) &= E\{V_p(t) V_p(t + \tau)\} \\ &= E\left\{\frac{A^2}{2} \cos [\omega_c t + \omega_c \tau + \mu(t + \tau) + \mu(\tau)]\right\} \\ &\quad + E\left\{\frac{A^2}{2} \cos [2\omega_c t + \omega_c \tau + \mu(t + \tau) + \mu(\tau)]\right\}. \end{aligned} \tag{5}$$

$E\{\bullet\}$ denotes the expected value operator. Since the noise has zero mean,

$$\begin{aligned} R(t) &= \frac{A^2}{2} E\{\cos [\mu(t + \tau) - \mu(t)]\} \cos \omega_c t \\ &= \frac{A^2}{2} \exp [R_\mu(t) - R_\mu(0)] \cos \omega_c t \end{aligned} \tag{6}$$

where $R_\mu(\bullet)$ is the autocorrelation function of the bandwidth limited noise. Therefore, the Fourier transform of the autocorrelation function which is the power spectrum is

$$S(\omega) = \frac{2}{\pi} \int_0^\infty \frac{A^2}{2} \exp [R_\mu(t) - R_\mu(0)] \cos \omega_c t \cos \omega t \, dt. \tag{7}$$

The frequency deviation, $\Delta\omega$, is the amount ω_i changes when $n(t)$ is at its peak value. Note $\Delta\omega$ also determines the total power of $n(t)$. Also define ω_n as the bandwidth or cutoff frequency for $n(t)$. The well-known rule of thumb known as Carson's Rule can be used for estimating the bandwidth, ω_{BW},

$$\omega_{BW} = 2\omega_n \left[1 + \frac{\Delta\omega}{\omega_n}\right]. \tag{8}$$

Carson's rule accurately predicts the bandwidth of a sinusoidal carrier continuously modulated with a band of Gaussian noise. In the FSK scheme presented here it can still be used to predict the bandwidth of V_p if the bandwidth of $n(t)$ is sufficiently below f_c. From (8) it can be seen that the bandwidth of the output voltage depends on the frequency band of the Gaussian noise as well as its amplitude. Carson's rule is merely meant to be a tool by which one can gain an estimate of the bandwidth of the noise modulated voltage. It also confirms the assumption that in order to eliminate substantial lower frequency content, the noise bandwidth must be limited. From Carson's rule, the noise source $n(t)$ must be bandwidth limited noise.

Besides simply controlling the bandwidth and amplitude of the noise, it may also be desired to vary other characteristics of the noise, depending on how machine-generated acoustic noise is affected. It is widely known that adding pre-emphasis causes the spectrum of the modulated voltage, $V_p(t)$, to roll-off at a faster rate than without pre-emphasis. Pre-emphasis implies that the amplitude of the noise linearly decreases over the frequency band. Without pre-emphasis the amplitude spectrum is

flat. The characteristics of the noise source can be precisely controlled using so-called "periodic" noise. This type of noise is simply the summation of desired frequency components with random phase. That is, for periodic band limited noise:

$$n_p(t) = \sum_{k \in S} N_p \sin(kt + \xi_k) \tag{9}$$

where S is the set of desired frequency components of n_p. N_p is the magnitude of the spectral components of the noise. N_p can be made a function of k to incorporate pre-emphasis. ξ_k is a uniformly distributed random number between 0 and 2π.

Since Carson's rule depends only on $\Delta\omega$, the minimum frequency component of $n(t)$ can be set to zero. In fact, it has been shown in [5] that the spectrum only changes slightly if the minimum frequency component of $n(t)$ is nonzero and is much less than the maximum frequency component.

IV. Implementation

A block diagram illustrating the implementation of the proposed random modulated sine-triangle PWM regulator is shown in Fig. 4. As can be seen from the figure, the random modulation is a very simple addition to an existing sine-triangle regulator. A sample and hold element is clocked at each peak of the triangle wave to hold the value of slope for the next segment.

The bandwidth limited noise source can be any Gaussian wide band white noise generator followed by a low pass filter, or a periodic noise generator. The white noise source can be obtained quite simply in a number of different manners, such as monolithic noise generator circuits, zener diode voltage, or a number of other semiconductor-based methods. However, the low pass filter requires a number of components including reactive elements. Also, the low pass filter should have a sharp cutoff in order to define the band of the output voltage as precisely as possible.

Fig. 5 shows another method for implementing the random carrier PWM regulator. In this approach the band limited noise generator is implemented using a lookup table, whose contents have been generated apriori, off line. That is, a large quantity of periodic random numbers can be computer generated and stored in some type of ROM. If the quantity of numbers is large, the repetition rate of the random numbers can be made large say, greater than one second, and will have no effect on the resulting voltage spectra. The output of the ROM is then sent through a digital to analog converter to get the slope of the triangle wave. With this approach the noise generator, hardware filters, and sample and hold element are not needed. In addition, since the noise is being generated off-line, any type of noise can be implemented, including periodic noise. It is then a simple matter to compare the acoustic noise generated by the machine as a function of the noise which modulates the carrier wave.

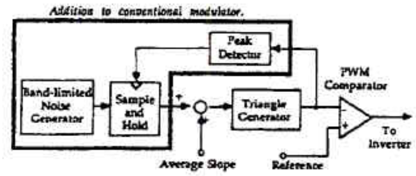


Fig. 4. Block diagram illustrating implementation of random carrier sinusoidal PWM regulator using analog noise generator.

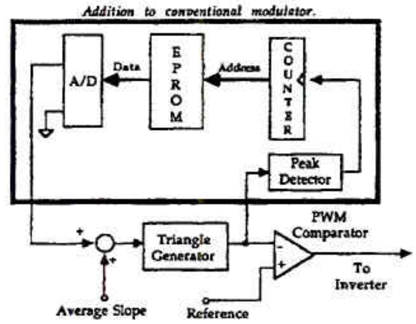


Fig. 5. Block diagram illustrating implementation of random carrier sinusoidal PWM regulator where noise source is generated digitally and stored in look-up table.

V. Experimental Results

An off-the-shelf 20-hp sinusoidal PWM inverter has been modified to include the random modulation scheme. The triangle carrier and the line switching function are shown in Fig. 6. Note that there is very little perceivable difference in the triangle wave with and without random modulation, since the instantaneous frequency is varying only slightly. Fig. 7 shows the harmonic spectrum of the Gaussian noise used to modulate the carrier. Note the effect of the added pre-emphasis which causes the lowest frequencies to be somewhat attenuated.

In all the results given, the output frequency is 17 Hz, the average switching frequency, f_c = 1 kHz; the cutoff frequency of the noise, f_n = 500 Hz for the band limited noise, and the peak frequency deviation, Δf = 300 Hz. The inverter is supplying an unloaded induction motor.

The line current at the induction machine terminals are shown in Fig. 8 with and without the random modulation. It is seen from this figure that the current is essentially unchanged due to the added random modulation. A linear scale plot of the line-line voltage spectra and the acoustic noise spectra are shown in Figs. 9 and 10 with and without random modulation. The acoustic noise spectra were measured with a sound pressure level meter, held in the same position for all measurements, with the microphone output connected to a spectrum analyzer. It is very important to note that the measured acoustic spectra are not meant to be absolute measurements of the acoustic noise

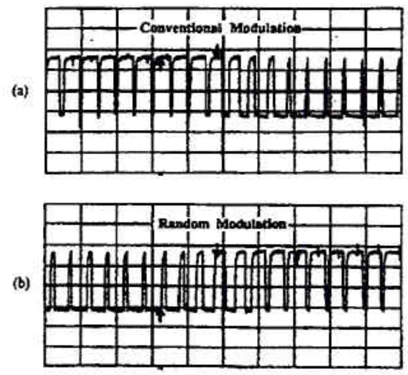


Fig. 6. Oscillograms of the pole switching functions. (a) Conventional sinusoidal modulation. (b) Randomly modulated sinusoidal PWM. Vert: 2 V/div. Horiz: 2 ms/div.

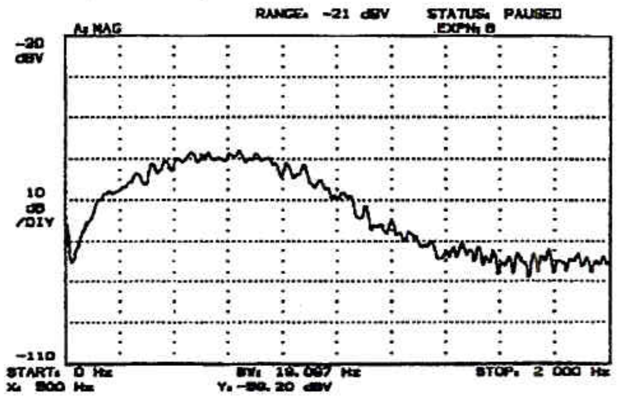


Fig. 7. Spectrum of noise used to modulate carrier waveform.

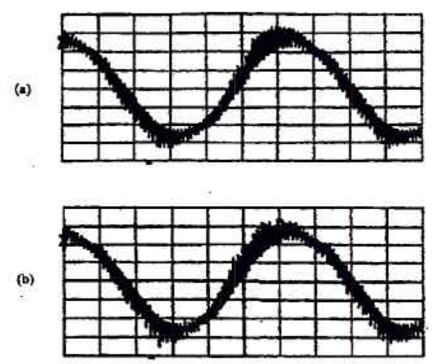


Fig. 8. Oscillograms of motor line current. (a) Conventional sinusoidal PWM. (b) Randomly modulated sinusoidal PWM. Vert: 10 A/div. Horiz: 10 ms/div.

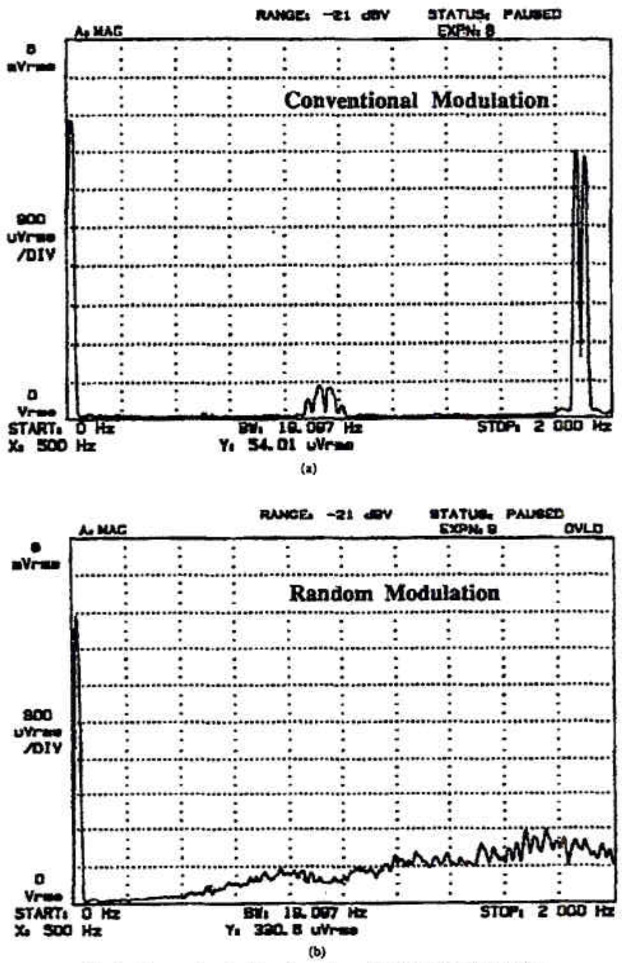


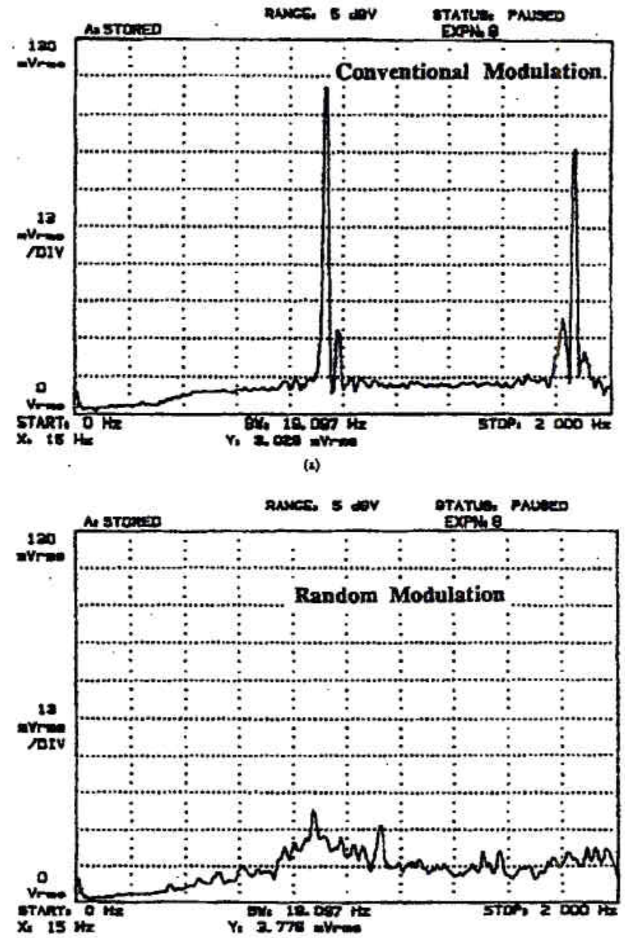
Fig. 9. Spectra of ac load line-line voltage. (a) Conventional sinusoidal PWM. (b) Randomly modulated sinusoidal PWM.

produced by the machine. Rather, figures 9 and 10 are meant to serve as relative comparisons showing the effect of random modulation. From these figures it can be seen that no appreciable energy at low frequencies (below half the switching frequency) is delivered to the machine due to the random modulation.

In Figs. 9 and 10, note the dramatic reduction in the magnitude of the individual harmonics near the switching frequency in the randomly modulated spectra. For the particular motor used in the experimental part of the work, the total (rms) sound energy generated by the machine is not reduced by the random modulation. However, the energy spectrum of the noise is now spread to create a more appealing, less annoying sound.

VI. Conclusion

Acoustic noise in electric machines is caused by the interaction of the fundamental and harmonic flux densities. The annoying whine that the motor produces can be avoided by eliminating distinct tones in the applied voltage. It is proposed that an effective method of spreading the spectral content of the applied voltage is by randomly modulating the triangle carrier in sinusoidal PWM. In this



(b)

Fig. 10. Spectra of acoustic noise generated by induction machine. (a) Conventional sinusoidal PWM. (b) Randomly modulated sinusoidal PWM.

way, the energy in the tones around the switching frequency is spread out with subsequent reduction in peak values. The random modulator maintains the advantages of sinusoidal PWM including constant average switching frequency, linear amplification, and real-time control. The instantaneous switching frequency variation is small and can be predetermined. With the proposed scheme, the total level of the acoustic noise emitted by the machine remains constant. The acoustic noise, however, is more pleasing to the ear since the noise is now random. Experimental results illustrate the absence of tones in the voltage applied to an induction machine and in the spectrum of the acoustic noise generated by the machine.

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